L6 Assessment 2

1 a) i) $\frac{1}{16} = 2^a$

$\frac{1}{2^4} = 2^{-4}$ $\quad \therefore a = -4$

ii) $\sqrt[3]{64} = 2^b$

$\sqrt[3]{2^6} = 2^2$ $\quad \therefore b = 2$

b) $\sqrt[3]{64} \times 2^x = \frac{1}{16}$

$2^2 \times 2^x = 2^{-4}$

$2^{2+x} = 2^{-4}$ $\quad \Rightarrow 2 + x = -4$

$x = -6$

2

$y = 3x^3 - 2x^2$ $\quad x = -1$

$\frac{dy}{dx} = 9x^2 - 4x$ $\quad y = 3(-1)^3 - 2(-1)^2 = -3 - 2 = -5$

$m_T = \left.\frac{dy}{dx}\right|_{x=-1} = 9(-1)^2 - 4(-1) = 9 + 4 = 13$

$\therefore m_N = \frac{-1}{13}$

$m = -\frac{1}{13}$ $(-1, -5)$ $\Rightarrow y + 5 = -\frac{1}{13}(x + 1)$

$13y + 65 = -x - 1$

$\therefore x + 13y + 66 = 0$

3

L_1 : $4y - 3x = 10$

$4y = 3x + 10$

$y = \frac{3}{4}x + \frac{10}{4}$ $\quad m_1 = \frac{3}{4}$

L_2 : $m_2 = \frac{8+1}{-1-5} = \frac{9}{-6} = -\frac{3}{2}$

Neither as $m_1 \neq m_2$ so not parallel.
$m_1 \times m_2 \neq -1$ so not perpendicular.

4 $x^2 + y^2 - 6x + 10y + 9 = 0$

a) i) $(x-3)^2 - 9 + (y+5)^2 - 25 + 9 = 0$

Centre is $(3, -5)$

ii) $(x-3)^2 + (y+5)^2 = 25 \quad \therefore r = 5$

b) $y = kx \qquad x^2 + (kx)^2 - 6x + 10kx + 9 = 0$

$x^2 + k^2x^2 - 6x + 10kx + 9 = 0$

Two distinct points of intersection $\Rightarrow$ two distinct solutions $\Rightarrow$ $b^2 - 4ac > 0$

$a = 1 + k^2$

$b = 10k - 6$

$c = 9$

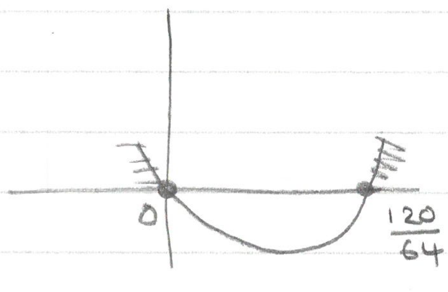
$(10k-6)^2 - 4(1+k^2)9 > 0$

$100k^2 - 120k + 36 - 36(1+k^2) > 0$

$100k^2 - 120k + \cancel{36} - \cancel{36} - 36k^2 > 0$

$64k^2 - 120k > 0$

$k(64k - 120) > 0$



{or use the calculator!}

$k < 0$ or $k > \frac{15}{8}$

5

$$f(x) = x^3$$
$$f(x+h) = (x+h)^3$$
$$f'(x) = \lim_{h \to 0} \left[\frac{(x+h)^3 - x^3}{h} \right]$$
$$= \lim_{h \to 0} \left[\frac{x^3 + 3x^2h + 3xh^2 + h^3 - x^3}{h} \right]$$
$$= \lim_{h \to 0} \left[\frac{3x^2h + 3xh^2 + h^3}{h} \right]$$
$$= \lim_{h \to 0} \left[\frac{h(3x^2 + 3xh + h^2)}{h} \right]$$
$$= \lim_{h \to 0} \left[3x^2 + 3xh + h^2 \right] = 3x^2$$

6 a) $L_1 : \; kx + (1-k)y = 5$

$$(1-k)y = 5 - kx$$
$$y = \frac{5-kx}{1-k} = \frac{-kx}{1-k} + \frac{5}{1-k}$$

$$\therefore m_1 = \frac{-k}{1-k}$$

b) $L_2 \; (1,4) \; (3,k)$

$$m_2 = \frac{k-4}{2}$$

c) i) $m_1 \times m_2 = -1 \Rightarrow \frac{-k}{1-k} \times \frac{k-4}{2} = -1$

$$\frac{-k(k-4)}{2(1-k)} = -1$$

$\rightarrow$

$$\frac{-k^2 + 4k}{2-2k} = -1$$

$$-k^2 + 4k = -2 + 2k$$

$$\Rightarrow k^2 - 2k - 2 = 0$$

c) ii) $k = 1 \pm \sqrt{3}$

7. $C(-2,6)$ $(10,11)$

a) $(x+2)^2 + (y-6)^2 = r^2$

Sub in $(10, 11)$

$(10+2)^2 + (11-6)^2 = r^2$

$144 + 25 = 169 \quad \therefore r^2 = 169$

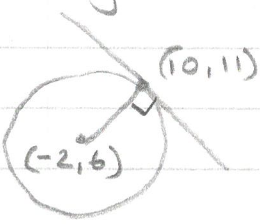
$(x+2)^2 + (y-6)^2 = 169$

Sub in $(10, 1)$

$(10+2)^2 + (1-6)^2$

$= 144 + 25 = 169 \quad \therefore$ the circle also passes through $(10,1)$.

b) Tangent at $(10,11)$:



$m_r = \frac{11-6}{10--2} = \frac{5}{12}$

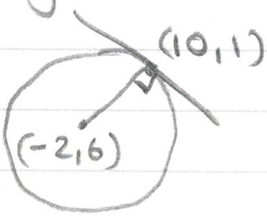
$\therefore m_T = -\frac{12}{5}$

$y - 11 = -\frac{12}{5}(x-10)$

$5y - 55 = -12x + 120$

$12x + 5y = 175$

Tangent at $(10,1)$:



$$m_r = \frac{1-6}{10--2} = -\frac{5}{12}$$

$$\therefore m_T = \frac{12}{5}$$

$$y-1 = \frac{12}{5}(x-10)$$

$$5y-5 = 12x-120$$

$$115 = 12x-5y$$

meets y-axis when $x=0$

$T_1: 12x+5y = 175$
$5y = 175$
$y = 35$

$T_2: 12x-5y = 115$
$-5y = 115$
$y = -23$

So P is $(0, 35)$
Q is $(0, -23)$
Hence PQ is $35+23 = 58$

(8)

$$y = 2x^3 + 6x^2 - 12x + 3$$

$$\frac{dy}{dx} = 6x^2 + 12x - 12$$

$$y = 1 + 60x - 6x^2$$

$$\frac{dy}{dx} = 60 - 12x$$

Chris says $6x^2 + 12x - 12 > 60 - 12x$

His claim is not true when $6x^2 + 12x - 12 \leqslant 60 - 12x$

$$6x^2 + 24x - 72 \leqslant 0$$

$$-6 \leqslant x \leqslant 2$$

Hence Chris is wrong when $-6 \leqslant x \leqslant 2$.